First International Tropospheric Airborne Measurement Evaluation Panel (TAbMEP) Meeting

Internal Estimate of Random Uncertainties

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Measurement Process

- Goal: A unified database of airborne measurements (species: gas, particulate, met, and radiative) with quantified uncertainty
- During a flight, measurements of species and parameters are obtained
  - Let each measurement be $x_i$
  - Measurement frequency depends on instrument
- Over a subspace (temporal and spatial) of a flight we might expect to measure a statistically stable level of the species, e.g. CO
  - mean of CO ($\mu_{CO}$) and natural variability of CO ($\sigma_{CO}$)
- Within this space we can further partition into time periods of length $t$
What do we actually measure?

- Goal is to estimate \((\mu_{CO}, \sigma_{CO})\)

  Let \(\bar{x}\) be the mean of multiple measurements over time \(t\)

  precision of \(\bar{x}\) is a function of time (no. of samples, \(n\))

Expectation or mean, denoted by \(E[\ ]\)

\[
E[\bar{x}] = E[x_i] = \mu_{CO} + \delta,
\]

where \(\delta\) is the instrument calibration bias

\(\delta\) is determined by comparison to a calibration standard (systematic)

\(\delta\) can be a function of the level of \(x\) (e.g. nonlinear)

Variance, denoted by \(V[\ ]\)

\[
V[x_i] = \sigma_{CO}^2 + \sigma_{E}^2,
\]

where \(\sigma_{CO}^2\) is the instrument precision (variability)

\(\sigma_{E}^2\) is the random variability of the instrument

\(\sigma_{E}^2\) can be estimated internally during the flight, under certain assumptions

\[
V[\bar{x}] = \left(\sigma_{CO}^2 + \sigma_{E}^2\right)/n
\]

Note total measurement uncertainty, \(TMU = \sqrt{\delta^2 + \sigma_{E}^2}\)
An Internal Estimate of Precision

- If we choose $t$ small enough, assume $\sigma_{CO}$ to be small relative to $\sigma_\varepsilon$
- Partition flight data into subsets of size $t$ and compute multiple estimates of $\sigma_\varepsilon$
- How long should $t$ be?
  - depends on the temporal and spatial variability of the species or parameter of interest
  - depends on instrument sampling rate
  - requires expert judgment
- To quantitatively test our judgment, we can plot $\sigma_\varepsilon$ estimates for varying $t$, and estimate the mean value
  - look for our estimate of $\sigma_\varepsilon$ to be robust over small range of $t$
  - If calibration precision is available (component of TMU), then we can compare to the internal estimate
**Internal Estimate Plot (Chen)**

- Note that the mode of the distribution is relatively constant over the range of $t$ from 40-120 seconds.
- If the standard deviation increases with longer times, it indicates the introduction of other components of variability (due to species).
  - Assumes shortest $t$ was chosen to exclude species variability.
How to combine multiple measurements?

- Consider two aircrafts, let $x$ and $y$ be the measurements from each

  \[ E[\bar{x}] = \mu_{CO} + \delta_1, \quad V[\bar{x}] = \frac{\sigma^2_{\varepsilon_1}}{n_1} \text{ (assumes } \sigma^2_{CO} = 0 \text{ over } t) \]
  \[ E[\bar{y}] = \mu_{CO} + \delta_2, \quad V[\bar{y}] = \frac{\sigma^2_{\varepsilon_2}}{n_2} \text{ (assumes } \sigma^2_{CO} = 0 \text{ over } t) \]

Let $z$ be the combined (unified) estimate of CO, consider a simple average

\[ z = \frac{\bar{x} + \bar{y}}{2} \]

\[ E[z] = \mu_{CO} + \frac{1}{2} \delta_1 + \frac{1}{2} \delta_2 \]

\[ V[z] = \sigma^2_z = \frac{1}{4n_1} \sigma^2_{\varepsilon_1} + \frac{1}{4n_2} \sigma^2_{\varepsilon_2} \text{, assumes } \text{cov}(x, y) = 0 \]

- Bias contribution from each instrument is reduced - still present
- Assumes equal weight (uncertainty) of measurements
Weighted Combination of Measurements

• If we have more information about the total measurement uncertainty, bias, and/or precision, consider a weighted average

\[ z = \frac{(w_1)\bar{x} + (w_2)\bar{y}}{(w_1 + w_2)} \]

if \( w_1 = w_2 \), it becomes a simple average

Let \( k_1 = w_1/(w_1 + w_2) \), \( k_2 = w_2/(w_1 + w_2) \)

\[ E[z] = \mu_{CO} + k_1\delta_1 + k_2\delta_2 \]

\[ V[z] = \sigma_z^2 = \frac{k_1^2}{n_1}\sigma_{\varepsilon_1}^2 + \frac{k_2^2}{n_2}\sigma_{\varepsilon_2}^2 \]

assumes \( \varepsilon \)'s are constants, \( \text{cov}(x, y) = 0 \), \( \sigma_{CO}^2 = 0 \) over \( t \)

• The weights could be based on calibration information or an internal estimate of precision as follows

\[ z = \frac{\left(\frac{1}{\sigma_{\varepsilon_1}^2}\right)\bar{x} + \left(\frac{1}{\sigma_{\varepsilon_2}^2}\right)\bar{y}}{\left(\frac{1}{\sigma_{\varepsilon_1}^2} + \frac{1}{\sigma_{\varepsilon_2}^2}\right)} \]
Summary

• Using knowledge of species temporal and spatial variation, allows for partitioning of the flight to isolate instrument precision

• A simple decomposition of measurements into components illustrates instrument uncertainty contributions

• Proposed a method an internal estimate of instrument precision from in-flight data, with a graphical test of validity

• Proposed a formulation of uncertainty estimates for
  – single aircraft campaigns
  – combining two (or more) instruments

• Method is generally applicable to all species and parameters
  – limitations may depend on available data